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Existence of Solutions of Anisotropic Elliptic Equations with Variable Exponents in Unbounded Domains

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Abstract—We consider a class of anisotropic elliptic differential equations of second order with divergent form and variable exponents. The corresponding elliptic operators are pseudo-monotone and coercive. We obtain solvability conditions for the Dirichlet problem in unbounded domains $\Omega \subset \mathbb{R}_n$, $n \geq 2$. The proof of existence of solutions is free of restrictions on growth of data for $|\mathrm{x}| \to \infty$.

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1. INTRODUCTION

Let Ω be a domain of space $\mathbb{R}^n = \{\mathrm{x} = (x_1, x_2, \ldots, x_n)\}$, $\Omega \subseteq \mathbb{R}^n$, $n \geq 2$. For anisotropic quasilinear elliptic equations of second order

$$\sum_{i=1}^{n} (a_i(\mathrm{x}, u, \nabla u))_{x_i} - a_0(\mathrm{x}, u, \nabla u) = 0, \quad \mathrm{x} \in \Omega; \tag{1.1}$$

we consider Dirichlet problem

$$u\Big|_{\partial\Omega} = 0. \tag{1.2}$$

We assume that functions $a_i(\mathrm{x}, s_0, s_1, \ldots, s_n)$ have power growth in variables s_i with variables $p_i(\mathrm{x}) \in (1, \infty)$, $i = 0, 1, \ldots, n$. The restrictions on functions $a_i(\mathrm{x}, s_0, \mathrm{s})$, $i = 0, 1, \ldots, n$, will be formulated in §3. As a simplest example we cite equation

$$\sum_{i=1}^{n} (|u_{x_i}|^{p_i(\mathrm{x})-2} u_{x_i})_{x_i} - |u|^{p_0(\mathrm{x})-2} u = \sum_{i=1}^{n} (\phi_i(\mathrm{x}))_{x_i} - \phi_0(\mathrm{x}). \tag{1.3}$$

In 1984 Brezis [1] studied equation

$$-\Delta u + |u|^{p_0-2} u = f(\mathrm{x}), \quad \mathrm{x} \in \mathbb{R}^n, \quad p_0 > 2,$$

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